Towards improving SAR search patterns by time-minimal paths

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Abstract

In this paper, we consider improving the standard search patterns under the effect of drift with application to the maritime and aeronautical search and rescue operations, basing on time-minimal paths being the solutions to the aircraft navigation problem.

Keywords: maritime Search and Rescue (SAR), search patterns, IAMSAR Manual, time-minimal trajectory, aircraft navigation problem.

1 Introduction

The standard search patterns are included and repeated in the subsequent editions of the International Aeronautical and Maritime Search and Rescue (IAMSAR) Manual, Vol. III "Mobile facilities", e.g. an expanding square, a sector search [1]. This publication is required to be up-to-date and carried onboard the ships worldwide by the International Convention for the Safety of Life at Sea (SOLAS) [2]. Each edition (the latest in 2019) is published jointly by the International Civil Aviation Organization (ICAO) and the International Maritime Organization (IMO). The search patterns are also applied in the softwares of the modern Electronic Chart Display and Information Systems (ECDIS) installed onboard the vessels and in the professional navigational simulators; for more details see, e.g., [3]. The simulations create a significant part of Maritime Education and Training (MET), which is emphasized by the International Association of Maritime Universities (IAMU).

It can be noticed that the effect of drift caused by the water currents or winds are sometimes neglected or oversimplified. This may increase a total time of Search and Rescue (SAR) operations, if the standard patterns are used routinely, especially with a ship's trajectory referred only to a fixed ground and carelessly in the problem under consideration. According to [1], on one hand, it is advisible for vessels to use dead reckoning navigation rather than more accurate navigational methods. On the other hand, accurate navigation is required. In particular, the first leg of a search model is usually oriented directly into the wind to minimize navigational errors. Furthermore, precise search patterns navigation using high-precision methods such as global satellite navigation systems will produce good patterns relative to the ocean bottom, but not relative to the drifting search object. Roughly speaking, the common search patterns have been selected for simplicity and effectiveness. The corresponding search plan includes estimating the most probable position of a distressed craft or survivors and taking drift effect into consideration. Today, the traditional methods and approaches including the human factors and convenience, straightforward procedures, and good seamanship are confronted the artificial intelligence, machine learning and sophisticated computational algorithms that create the base for the future autonomous ships. We recall that IMO wants to ensure that the regulatory framework for Maritime Autonomous Surface Ships (MASS) keeps pace with technological developments that are rapidly evolving [4]. Furthermore, supporting search and rescue missions with Unmanned Aerial Vehicles (UAVs) is considered for various applications; see, for instance, [5, 6] in this regards.

Having in mind all the above factors, we ask whether the standard search models used in the SAR operations can be refined in order to decrease time of search, which matters decidedly. In our concept we aim at making use of the time-minimal paths, taking into account the acting perturbations (currents, streams, winds) *in situ* in addition. Such optimal paths are the solutions of the planar aircraft (Zermelo's) navigation problem. The problem is to find an optimal steering in the presence of perturbing vector fields so that the travel time from one position to another is minimal [7, 8]. Furthermore, another question which arises here is: in which types of scenarios, i.e. currents, are the standard search patterns based on the straight line legs optimal in the sense of time (at least locally)? In general, the currents depend on position and/or time as well as the effect of drift may vary on both the searching and searched objects. By this paper our objective is also to present some remarks on the IAMSAR Manual (Vol. III) in order to stimulate a discussion about its recommendations during the IAMU AGA 21 Conference, and to pay attention of the audience to somehow neglected impact of drift on the standard patterns used in some SAR missions at sea. As a consequence, this covers automated search pattern implementation in electronic navigation systems as well as a debate on efficient application of the improved and new search patterns in the future.

2 Standard search patterns applied in maritime and aeronautical SAR operations

First, we recall the specific search patterns which are applied in Search and Rescue operations and described in the IAMSAR Manual. The Manual provides guidelines for a common aviation and maritime approach to organizing and providing search and rescue services. The essential criterion in the SAR operations is time. The standard patterns

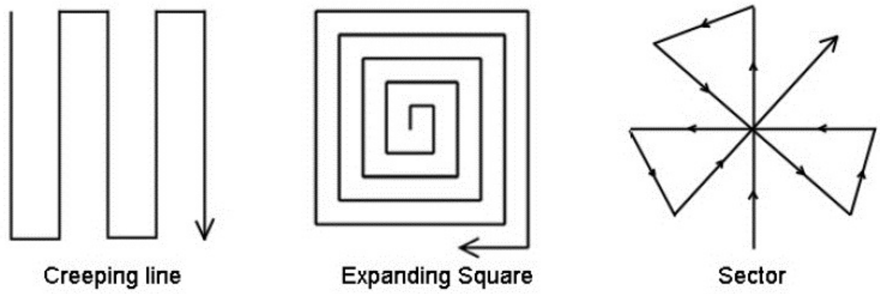

Figure 1: The standard search patterns as per the IAMSAR Manual: creeping line, expanding square and sector search [1].

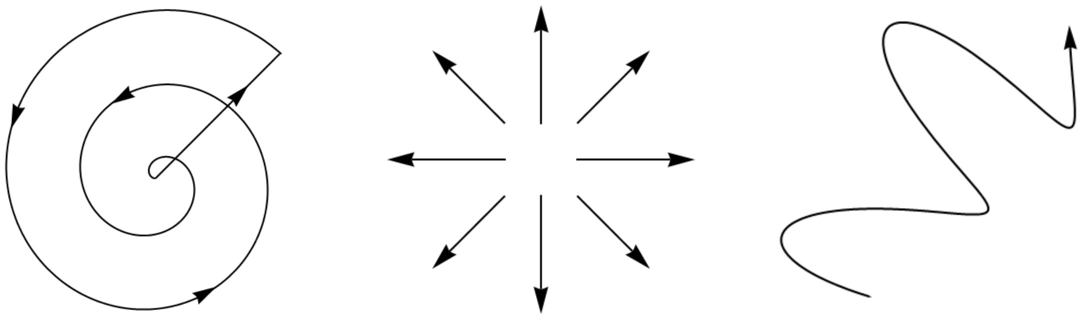

Figure 2: Other possible search methods: spiral, radial, random.

are applied when the accurate position of a searched object is unknown. The objective is to search the area efficiently (area coverage) and in the shortest time possible (time optimization). Therefore, a point of the highest probability (datum) which represents the centre point of a search space is determined first. In general, the common models are reliable and give the best result in particular circumstances. However, there is a need for an alternative approach in some cases. We remark that this is also the appropriate field for applying the theory of reliability and probability including notion of concentration ellipse as in the study of uncertainties and error analysis. In our investigation we focus on the paths of least time which could also be applied by searching crafts in order to optimize the operation, for instance, with support of the unmanned aerial vehicles.

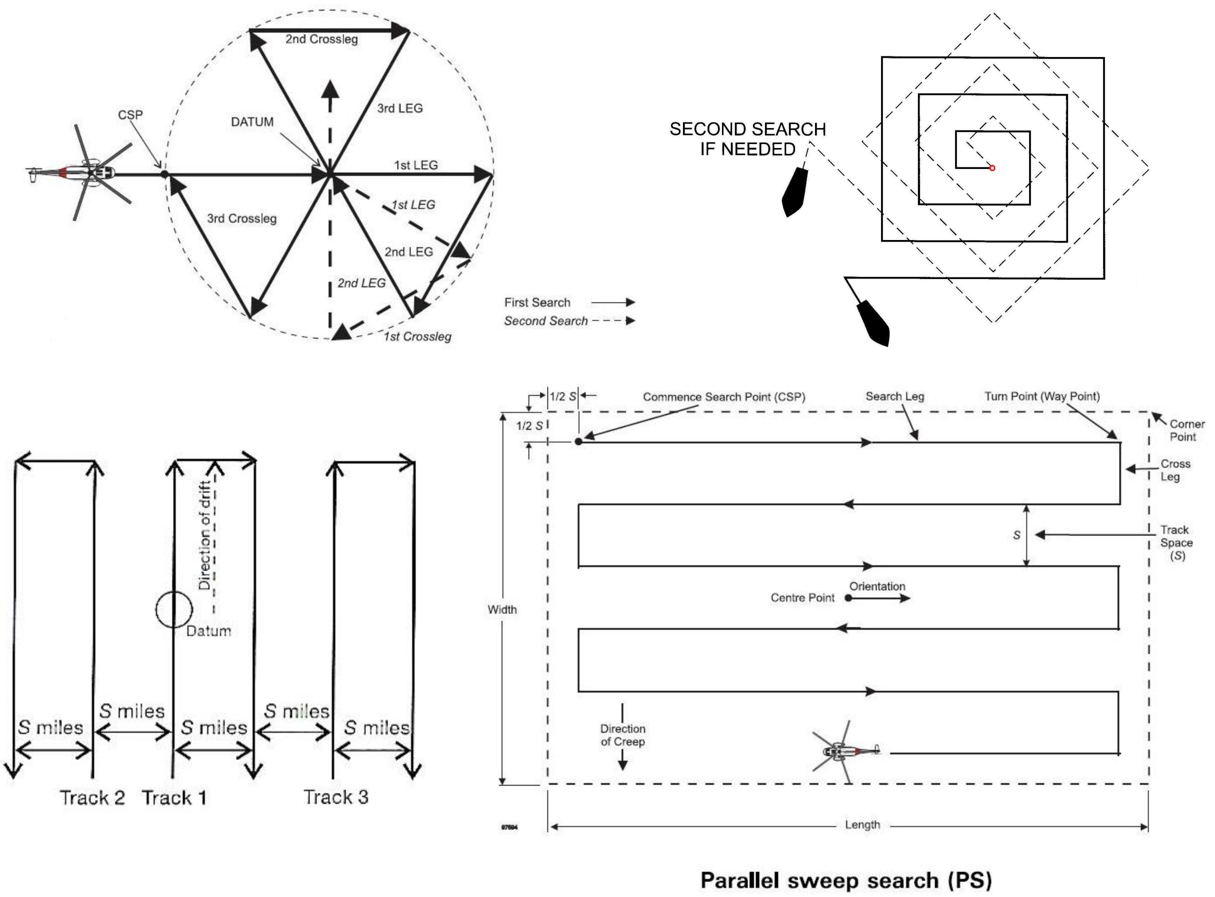

Figure 3: The SAR patterns in the aeronautical applications: sector search (top left), expanding square (top right), parallel track search - 3 ships (bottom left), creeping line (coordinate vessel-aircraft pattern; bottom right) [1].

It can be observed that the search models are created and required to be used routinely, somehow neglecting the influence of acting wind/current field in the meaning of optimization. The standard methods of search are based on the following patterns: expanding square, creeping line (coordinate vessel-aircraft pattern), sector and parallel search. They are presented in Figure 1 and Figure 3. Other possible methods are added in Figure 2. Planning the search operations, the standard paths are followed in the air and marine applications what self-explanatory Figure 3 shows. In fact, a search path can be flown/sailed in an active or passive way. The former means that we correct the route to execute search pattern over a fixed ground. In turn, we let the craft to be drifted continuously by an acting perturbation in the latter, so we follow the search pattern just in relation to air/water. With all of the above in mind we ask whether the search paths (entire or their segments) could be based on time-minimal paths in order to fulfill the time criterion more efficiently.

3 Simulations including action of constant currents

In this part of our investigation we used several softwares included in the professional navigational simulators at the Faculty of Navigation of the Gdynia Maritime University, i.e. K-Bridge ECDIS (software version 7.1.5.78) and Planning Station of the K-Bridge Navigation Software and Polaris from Kongsberg Maritime AS, the navigation information system Navi-Sailor 4000 ECDIS integrated to the navigational simulator Navi-Trainer Professional 5000 by Transas Technologies Ltd. (currently, Wärtsilä) as well as NACOS Platinum by L3 Marine Systems (currently, Wärtsilä). These are the ones of the worldwide leaders in developing a wide range of IT solutions for the marine industry and promote their own concepts of e-Navigation. Now we consider some examples referring to real world applications. To begin with, we activate the search paths with the scenario without perturbation (a calm sea model). The generated routes based on expanding square, sector search and parallel tracks in case of one searching ship are presented in Figure 4. We assume a constant speed of the craft sailing (or flying at constant altitude) in a two-dimensional area. In practice, a search speed means a maximum speed of a ship which is possible to be kept in the real conditions. Note that for $n \geq 2$ searching ships this means the highest speed of the slowest ship in a group. The simulated trajectories correspond to the standard patterns.

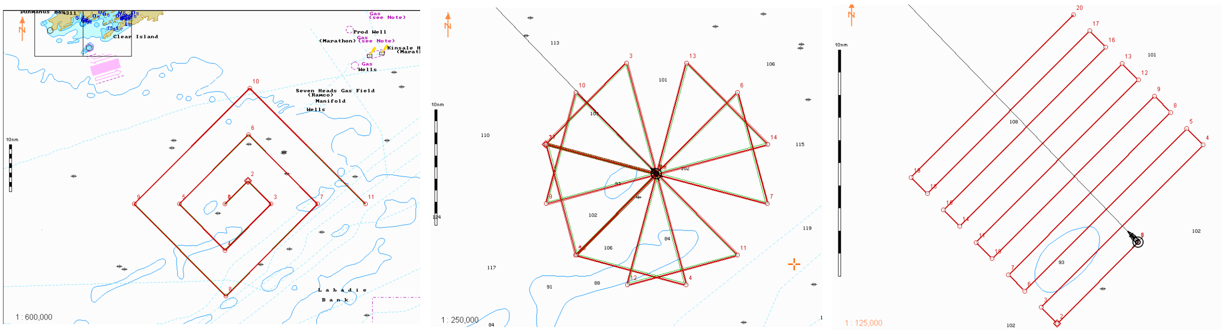

Figure 4: The simulated search paths based on expanding square (left), sector search (middle) and parallel tracks (right) in the absence of current (wind) field. The initial position of search (datum) is (50°50.0'N, 008°50.0'W) and the search speed is $|u| = 10$ kn.

Next, we introduce some perturbations with the same datum in the way it is possible to be set up in the system (ECDIS). In fact, this is not technically feasible to take into consideration any drift in the K-Bridge and NACOS Platinum simulators. Therefore, the attached figures including the modified search models with constant perturbation come from the second simulator, i.e. Navi-Sailor 4000. The current/wind is defined by two parameters, i.e. "drift" and "set" stand for direction and speed, respectively. We set the following conditions in the simulations. In the expanding square model presented in Figure 5 the speed of the ship equals 10 kn, the perturbation is determined by the pairs of drift and set as follows: (135°, 2.0 kn), (270°, 2.0 kn), (180°, 1.0 kn). The commence search point is given by the geographical coordinates (50°50.0'N, 008°50.0'W), the number of legs equals 10, the starting leg length is 7 Nm, and the search pattern heading equals 315°. Similarly, we generate the pattern for the sector search. The solutions obtained in the navigational simulator are given in Figure 6. The number of sectors is 7, the search radius equals 10 Nm and the turn angle equals 30°. For comparison, the analogous results are also presented for the model of parallel tracks under the action of the same perturbations, with track spacing 1 Nm (Figure 7).

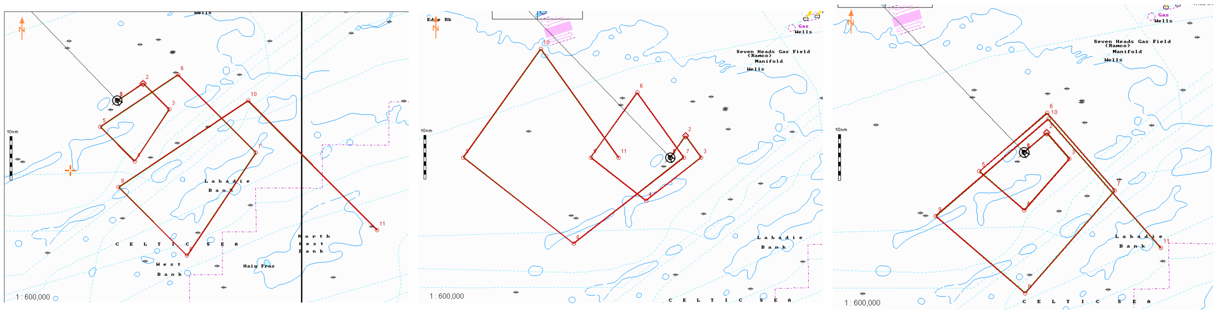

Figure 5: The simulated search paths in the model of expanding square, under acting current (wind) field with set of 2.0 kn and drift: 135° (left), 270° (middle) as well as 1.0 kn and 180° (right); the search speed is $|u| = 10$ kn.

Accordingly, the Euclidean plane with just steady current is used in order to generate the search paths. We note that, in fact, only constant currents (if any) are applied in the professional simulators as well as the corresponding devices onboard marine ships (aircrafts), since the same software is implemented. Moreover, the drift effect is assumed here to be always the same for the searching and searched objects. Clearly, in each of the simulated scenarios, if we subtract the drift vector given as a linear function of time, then we obtain the standard search patterns as illustrated initially in Figure 4. Hence, in reference to a flowing medium (water, air) the standard model is followed continuously. However, the search paths over fixed ground are then modified by the vectors of drift as presented in the subsequent figures. Nevertheless, it can be observed that many students of marine navigation and practitioners (mates) follow only the search routes generated in ECDIS (with respect to a ground) during the simulations of SAR exercises, and irrespectively of the acting currents. Such approach may increase time of entire operation considerably.

Due to some practical reasons the simplified approach is followed routinely in reality. However, taking into consideration the state-of-art in positioning, modelling, tracking and optimal control technology in the context of robotic sailing or drone (UAV) piloting, such approach becomes oversimplified nowadays because the models can be improved. This remark plays a relevant role, if the perturbations are varying in space and/or time, so like the real ones are in fact.

4 Improved navigation formula for computing time-optimal paths in flow fields of an arbitrary force

In [7] the aircraft navigation problem was emphasized in optimal control theory and approximate solutions, considering in many different contexts, i.e. controllability, simplest

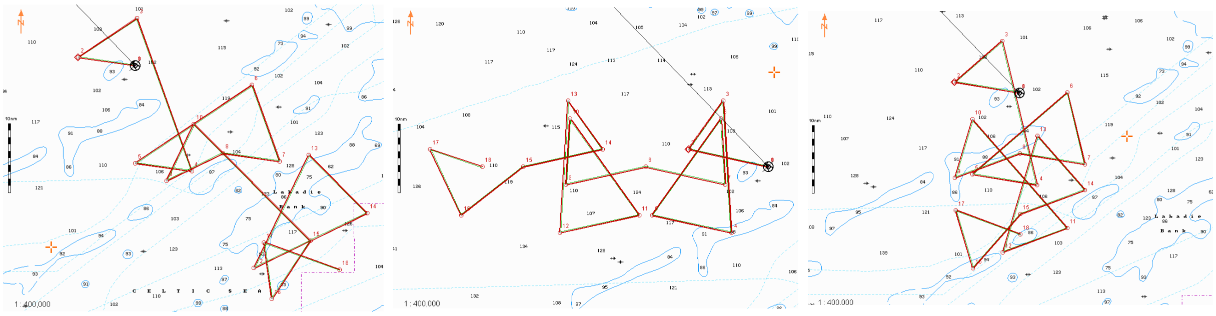

Figure 6: The simulated search paths in the model of sector search, under acting current (wind) field with set of 2.0 kn and drift: 135° (left), 270° (middle) as well as 1.0 kn and 180° (right); the search speed is $|u| = 10$ kn.

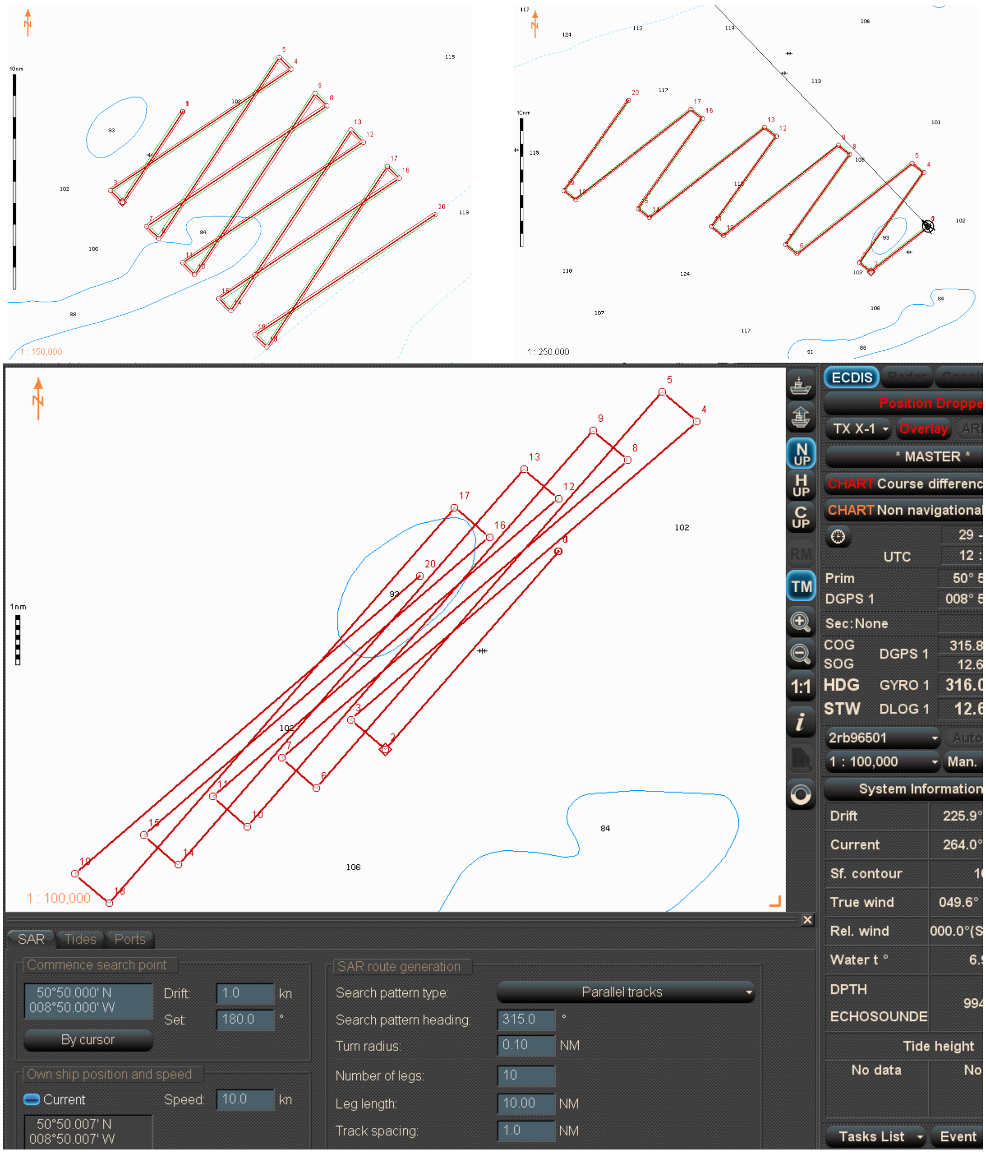

Figure 7: The simulated search paths in the model of parallel tracks, under acting current (wind) field with set of 2.0 kn and drift: 135° (top left), 270° (top right) as well as 1.0 kn and 180° (bottom); the search speed is $|u| = 10$ kn.

optimal control problems, fixed final time (first and second differentials), free final time, parameters, free initial time and states as well as approximate solutions of optimal control problems. The simplified form is stated as follows: find the control (the steering angle) of a constant speed aircraft (ship) in a crosswind flying (sailing) from one point to another that minimizes the final time. The problem can be converted into a "fixed final time" with the aim to minimize the performance index by introducing the transformation time or considering the problem that has no states. The above considerations were presented on the Euclidean plane with the use of the Hamiltonian formalism; see also [9]. With reference to the genesis of the aircraft navigation problem, we have revisited it recently. Namely, we presented some generalized and extended results regarding a backgorund space (an arbitrary surface and more generally, a conformally flat Riemannian manifold), and a varying speed in the presence of currents which depend on space and time [10, 11, 12]. In particular, these results include a condition for point-to-point time-optimal navigation in the Euclidean plane in which the craft is modelled as a particle moving at variable speed relative to the surrounding current/wind.

The aircarft (Zermelo) navigation problem in $\mathbb{R}^2$ became classical in optimal control theory and often present in various real-world applications; see [7, 13, 14, 15, 16, 17, 18, 19, 20]. When the wind is not everywhere zero, time can often be reduced by deviating from the geodesic route to take advantage of weaker headwinds or stronger tail winds (using aviation terminology), the greater length of the route being more than compensated by the increase in ground speed. Analogous scenario can be applied to marine setting with water currents or streams. More precisely, we have the following result

Theorem 4.1. *[11] Let (M,h) be a Riemannian manifold of dimension 2 with the metric given by $h_{ij}(x) = S^{-2}(x)\delta_{ij}, i,j = 1,2$ and (h,W,f) be the navigation data. The time-optimal paths in arbitrary wind on (M,h) are determined by the following ordinary differential equation*

$$\begin{aligned}\dot{\varphi}(t) = &-\frac{\partial W^1}{\partial x^2}\cos^2\varphi + \left(\frac{\partial W^1}{\partial x^1} - \frac{\partial W^2}{\partial x^2}\right)\sin\varphi\cos\varphi + \frac{\partial W^2}{\partial x^1}\sin^2\varphi \\ &+ S\frac{\partial f}{\partial x^1}\sin\varphi - S\frac{\partial f}{\partial x^2}\cos\varphi + f\frac{\partial S}{\partial x^1}\sin\varphi - f\frac{\partial S}{\partial x^2}\cos\varphi,\end{aligned} \tag{1}$$

and the equations of motion $\dot{x}^1 = W^1 + fS\cos\varphi$, $\dot{x}^2 = W^2 + fS\sin\varphi$, if $S(fS + W^1\cos\varphi + W^2\sin\varphi) \neq 0$ and $\varphi \in [0, 360°)$ is a heading angle.

The notation applied above is: $W^i(x,t)$ - wind (current) components, $f(x,t)$ - a self-speed of a ship, $S(x)$ - a conformal factor and δ_{ij} is the Kronecker delta; for more details, see [11]. The condition for optimal steering (1) can be expressed in a slightly different form using the definition of a heading in navigation which is taken as positive clockwise from north (φ_{nav}), not as positive running anticlockwise from east (φ) which is usual convention in mathematics. This then yields

$$\begin{aligned}\dot{\varphi}_{nav}(t) = &\frac{\partial W^1}{\partial x^2}\sin^2\varphi_{nav} + \left(\frac{\partial W^2}{\partial x^2} - \frac{\partial W^1}{\partial x^1}\right)\sin\varphi_{nav}\cos\varphi_{nav} - \frac{\partial W^2}{\partial x^1}\cos^2\varphi_{nav} \\ &- \left(S\frac{\partial f}{\partial x^1} + f\frac{\partial S}{\partial x^1}\right)\cos\varphi_{nav} + \left(S\frac{\partial f}{\partial x^2} + f\frac{\partial S}{\partial x^2}\right)\sin\varphi_{nav}.\end{aligned} \tag{2}$$

It is observed that if $f = f(x)$ or $W = W(x)$, then the optimality condition has still the same form. Namely, time-dependence does not change the resulting formula.

For the convenience of the reader and simplicity, we refer to the planar (Euclidean) setting. Remark that the optimality condition for the plane was applied in various investigations concerning optimal aircraft routing in general wind fields [21] or path planning for unmanned aerial vehicles in steady uniform winds [22]. If $h_{ij} = \delta_{ij}$, $i,j = 1,2$, so $S = 1$, then Eq. (1) is reduced to the condition for optimal navigation in $\mathbb{R}^2$, i.e.

$$\dot{\varphi}(t) = -\frac{\partial W^1}{\partial x^2}\cos^2\varphi + \left(\frac{\partial W^1}{\partial x^1} - \frac{\partial W^2}{\partial x^2}\right)\sin\varphi\cos\varphi + \frac{\partial W^2}{\partial x^1}\sin^2\varphi + \frac{\partial f}{\partial x^1}\sin\varphi - \frac{\partial f}{\partial x^2}\cos\varphi. \tag{3}$$

We note that Eq. (3) can be presented in a different form including the components of the resulting velocity v^i instead the components of a current W, i.e., substituting $W^1 = v^1 - f\cos\varphi$, $W^2 = v^2 - f\sin\varphi$, where $f = f(t, x^1, x^2)$. We thus have

$$\dot{\varphi}(t) = -\frac{\partial v^1}{\partial x^2}\cos^2\varphi + \left(\frac{\partial v^1}{\partial x^1} - \frac{\partial v^2}{\partial x^2}\right)\sin\varphi\cos\varphi + \frac{\partial v^2}{\partial x^1}\sin^2\varphi, \tag{4}$$

where $v^1 = \dot{x}^1$, $v^2 = \dot{x}^2$. In particular, if $S = 1$ and $f = 1$, then Eq. (1) leads to the classic navigation formula obtained initially in $\mathbb{R}^2$ [8].

The above optimality condition (3) can be expressed concisely. Namely, if at a given point of the optimal paths the rectangular coordinate system is chosen so that the x^1-axis coincides with the ship's heading, then Eq. (3) with variable self-speed reduces to $\dot{\varphi} = -\frac{\partial W^1}{\partial x^2} - \frac{\partial f}{\partial x^2}$. Equivalently, $\dot{\varphi} = -\frac{\partial v^1}{\partial x^2}$ by Eq. (4). This with $f = 1$ yields the classic navigation formula on the Euclidean plane in the shortest form $\dot{\varphi} = -\frac{\partial W^1}{\partial x^2}$[1]. Summarizing, the concise form of the condition for optimal navigation (1), with the local coordinate system referred to a ship, reads

$$\dot{\varphi} = -\frac{\partial(W^1 + fS)}{\partial x^2} \quad \text{or} \quad \dot{\varphi} = -\frac{\partial W^1}{\partial x^2} - S\frac{\partial f}{\partial x^2} - f\frac{\partial S}{\partial x^2}. \tag{5}$$

Using the condition for optimality, the time-minimal paths joining the waypoints that characterize the standard search patterns can be found. It is shown that they differ from the straight line paths [24]. Thus, some local modifications to the standard search patterns are introduced in such a way that a search pattern similar to the original one is followed in minimal time by a particle that represents the ship/ aircraft, and the coverage of the search area is ensured.

5 Exemplary modifications of search models based on time-minimal paths

The time fronts ($t = 1$, dashed colors) of the straight line and least time motions are first compared during the passage in windy conditions so that the whole area is fully covered (searched). The time-minimal paths (solid red) simulating the trajectories in a unit disc starting from the origin, with the heading increments $\triangle\varphi_0 = 10°$, under the linear wind field $W = (y, 0)$ in the Cartesian coordinates system $x0y$ are illustrated in Figure 8. It is clear that in the absence of current the optimal paths are described by straight rays (black arrows) coming from the centre, if the search speeds are constant. In the presence of wind field the space can be fully covered alternatively by curved time-optimal paths after suitable adjusting the initial headings φ_0, which determine uniquely the corresponding minimizing paths. Moreover, the travel time is saved in this case.

In contrast to the search paths created in the software of the navigational simulator we continue with the model including the perturbation W which depends on position. The standard patterns are combined with the time-optimal paths which are represented

[1]"The helm has always to be turned to that side in which the wind component acting against the steering direction increases" [23, p. 116]

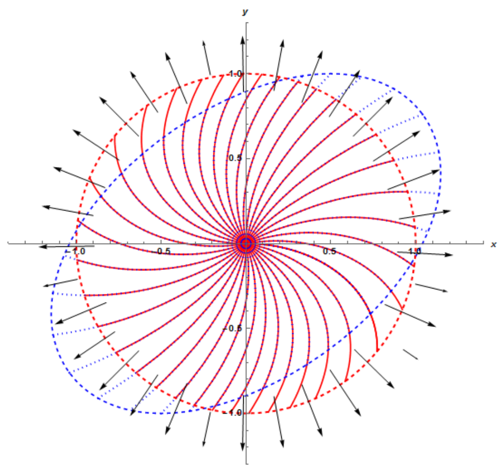

Figure 8: The time-minimal paths (solid red) in a unit disc starting from the origin, with the increments $\triangle\varphi_0 = 10°$ and the time front ($t = 1$, dashed blue), under the linear (shear) wind (or current) $W = (y, 0)$ [24, 25].

by the local solutions to the aircraft navigation problem. Regarding computations we apply the system consisting of the corresponding equations of motion and the optimality condition presented in Section 4. For simplicity, let W be given by the river-type flow, i.e. $W = (f(y), 0)$. We now consider the piecewise time-minimal paths connecting the fixed waypoints of the route defined in the standard models, where the directions of the straight search paths change at right angles. Thus, the concept is to make use of the current in order to increase the resulting speed, and not to follow the fixed standard pattern without regard for the type and properties of acting currents. Roughly speaking, if possible, we aim at avoiding to sail against perturbation routinely.

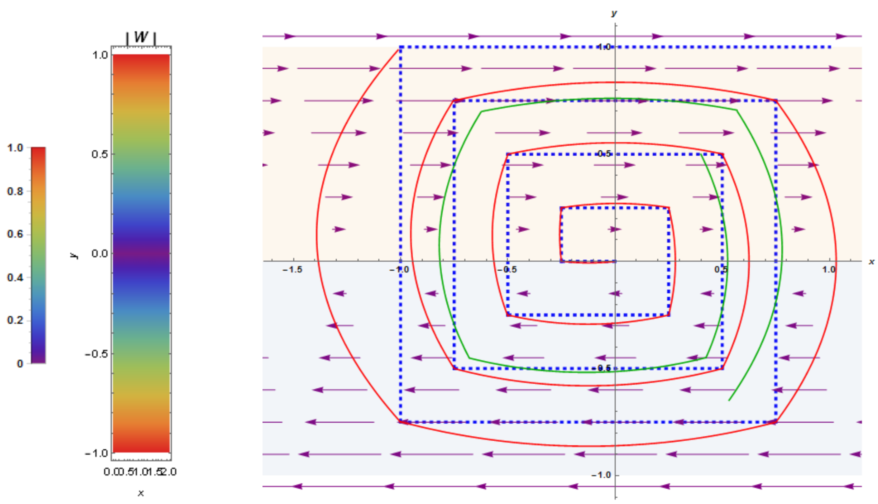

Figure 9: The expanding square model (dotted blue) modified by the piecewise-time-optimal legs (red and green) under the shear (linear) current field $W = (y, 0)$. Middle: the current speed (to scale) [24, 25].

We proceed by considering expanding square and, for simplicity but without loss of the general concept, the weak linear vector field $W = (y, 0)$. In Figure 9 we show the standard expanding square of given starting leg length which determines track spacing ε^* in the entire model and oriented so that the horizontal legs are parallel to the flow. Thus, the fixed waypoints are determined and represent the consecutive startpoints/endpoints connected by the time-minimal paths (solid red). Since time of passage is shorter in each

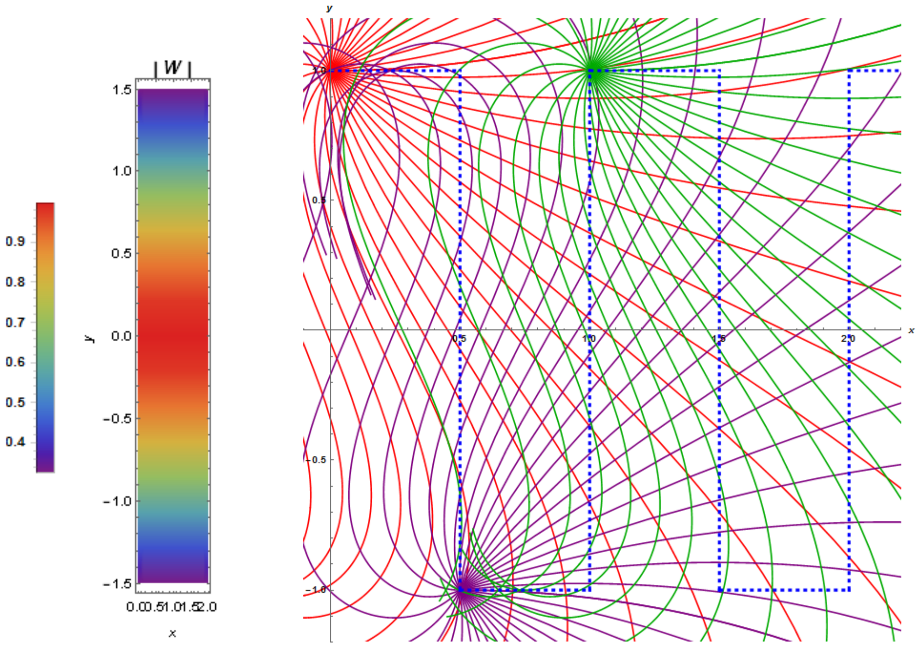

Figure 10: The creeping line search modified by the time-minimal legs starting from the fixed waypoints determined by the standard pattern, under the Gaussian wind field $W = (\frac{5}{2}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}y^2}, 0)$; $\triangle\varphi_0 = 10°$. Middle: the current (wind) speed (to scale) [24, 25].

leg in comparison to the corresponding straight legs of the standard pattern, the total time along the new modified paths is expected to be decreased. Obviously, we require that the search is efficient, that is, the maximum distance ε between the points of the searched space and the search path also needs to be taken into consideration. Namely, we require that the search is *complete*, i.e.

$$\forall \;\; A \in \mathcal{D} \;\; \exists \;\; \tilde{A} \in \Gamma : \;\; d(A, \tilde{A}) \leq \varepsilon, \tag{6}$$

where a sub-area $\mathcal{D} \subset M$ of a metric space (M, d) is a search space, $\Gamma = \bigcup_i \gamma_i$ is a search path, γ_i represents a time-optimal leg and $\varepsilon \geq 0$ is a fixed parameter. The condition implies that there are no omitted zones if the time is free and a ship follows the time-minimal legs in a complete search. Namely, the points of searched area should be close enough to at least one path. Hence, the solutions to the navigation problem in compliance with the condition (6) can be applied to the problem of search. The particular application depends on the initial conditions, type of perturbation and preset parameter ε. Each leg γ_i guarantees local optimality in the connections between the intermediate waypoints. However, we recall that the key goal is to minimize the total time t_c of a complete search. Otherwise, the models based on the piecewise time-optimal search path Γ which can be represented by the suitable solutions to the navigation problem might not state for the final time-optimal solution of the search problem. So far, we aimed at showing the potential application of (piecewise) time-optimal paths, where it is reasonable in order to minimize the total time of search without exceeding required value of ε. For that reason previously fixed waypoints are now translated such that the obtained new least time connections (solid green in Figure 9) fulfill the condition for the maximum distance between the points of searched space and search paths.

Next, we consider the creeping line search in the presence of the specific Gaussian flow $W = (\frac{5}{2}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}y^2}, 0)$. This is shown in Figure 10. In analogous way the standard paths (dotted blue) are modified with the use of time-minimal legs starting from the fixed waypoints obtained by the common pattern. The charts of the color-coded paths are created and they cover the area in the presence of acting current under consideration. The curves start from determined fixed points and represent the quickest connections. However, in the search problem the additional conditions are included, e.g., ε or restricted domain, in the context of free or fixed final time problems which are therefore of interest of optimal control. In each scenario the conditions modify the final search model. Thus, the situation-dependent approach is required. The objective is to make use of the minimum time legs to decrease the total time of searching the whole area or to maximize the area which can be searched in the limited fixed time. Beside combining the solutions to the navigation problem with the standard models it could be reasonable in the presence of some perturbations to omit the common paths completely. This means to follow non-standard search models configured with the time-optimal legs and rearranged wypoints without any references to the standard patterns.

To our knowledge, there are no effective solutions (including optimality and feasibility) dedicated to the search patterns in a two-dimensional area, taking into coniderationa drift effect which minimize the total time of a complete search (full coverage). Thus, we can formulate the challenging task for the future study as follows: to create a feasible algorithm so that the total time of a complete search is minimized, under the action of wind field varying in space and time in case of

1. one ship (UAV) searching alone; and
2. $n \geq 2$ ships (UAVs) searching simultaneously and in coordination.

The practical interest in the above open problems refers to the current need for efficiencies in performance of maritime and aerospace vehicles (drones) as well as systems of vehicles to meet advanced operations of great importance. Furthermore, it should be emphasized that improving the search models requires that the search is efficient (both the global time minimum and area coverage), not just "optimal" in the meaning of following time-optimal legs locally.

6 Discussion and conclusions

We focused on refinement of the standard SAR patterns under the effect of drift by time-optimal paths. Such legs connecting the consecutive waypoints of a search route are not necessarily straight lines, however they minimize travel time locally between the positions in the search area. The formally recommended search patterns have been selected for simplicity and effectiveness, but they may not be efficient in various situations with action of the perturbing flow fields. The standard patterns are reliable and give good result in particular circumstances. However, there is a need for an alternative approach in some cases and examining different methods of shortening search duration. The behaviour of the trajectories of least time depend on the acting currents or winds significantly. The advantage of the proposed approach is more visible when the ratio of current speed to ship's speed increases, and the varying currents depend on position or the effect of drift is different for a searching ship and a searched object. Taking into account information on current direction and speed in situ which are more and more reliable and available today, the new patterns can save time and effort that are wasted for searching low probability areas as well as make use of the acting flows.

The idea was to make use of time-minimal paths in the problem of search when this is reasonable, i.e. reduces the total (global) time of a search. With the condition for a complete search in mind we aimed at decreasing time of search in this preliminary modelling in comparison to the existing methods, where the standard patterns are followed. They are routinely used without considering the types of current (wind) fields and initial conditions. Only the standard models are required formally to be used in real-world aeronautical applications; see [1]. This motivated us to revisit these patterns and focus on the reasons which enforce them to be followed in the scenarios under the action of a flow field. In particular, the study shows that it is not necessarily efficient to orient the search model so that a starting leg is parallel to current (wind) as it is routinely assumed. Since main criterion is time under complete search condition, combining the standard models with time-minimal paths or creating the new models based solely on such trajectories in the presence of perturbation can lead to higher efficiency. This calls for further study in order to adapt to the practical capabilities and requirements. Note that the current technology enables to implement the models based on time-minimal paths, for instance, in route planning and monitoring referring to the drone aerial survey and patrolling fixed zone, robotic sailing, weather routing combined with the numerical weather prediction models. Concerning implementations the applied simplifications admitting only constant perturbation as in the modern navigational software can be improved by considering stationary currents with the use of time-optimal solutions similarly to the presented examples. The standard search patterns may become inefficient with respect to the criterion of time. This fact gives meaningful opportunity to apply more advanced models due to the essential time reduction in the scenarios with a (relatively stronger) drift effect.

Regarding Maritime Education and Training it is recommended that more attention should be paid to the generated and followed search patterns with reference to the fixed ground in ECDIS, e.g. using GPS during simulations of SAR operations in the areas, where the drift effect is observed. In particular, following the standard patterns plotted only over ground and not with respect to the flowing water may cause increasing time of search and rescue mission instead of decreasing. Furthermore, a more general formulation of the problem should be considered in the further study in which the important aspect is to ensure the coverage of the zone to be explored and the pattern followed in the search is not fixed in advance and is determined by solving the problem. For instance, the coverage constraints could be enforced using waypoints which, depending on the current/wind field, are reached in a different order and at different time instants. Another aspect to be considered is the fact that in aeronautical search and rescue operations minimization (or at least estimation) of the fuel consumption is an important aspect. This means that more realistic models of current/ wind and ships should be applied.

Acknowledgments

This work was partially supported by the Gdynia Maritime University project reference WN/PI/2021/01.

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